

Mining of Real-world Hypergraphs: Concepts, Patterns, and Generators

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CIKM'22

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- 0. Introduction
- 1. Static Structural Patterns
- 2. Dynamic Structural Patterns
- 3. Generative Models

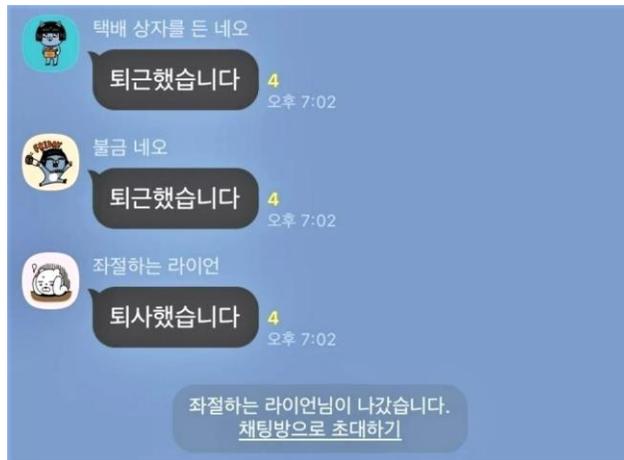
Mining of Real-world Hypergraphs: Concepts, Patterns, and Generators

- 0. Introduction 전반적인 개념 소개 및 설명
- 1. Static Structural Patterns 구조적 패턴을 통한 Hypergraph 분석
- ~~2. Dynamic Structural Patterns 구조적 패턴을 통한 Time-evolving Hypergraph 분석~~
- 3. Generative Models Static, ~~Dynamic~~ Hypergraph 생성 모델

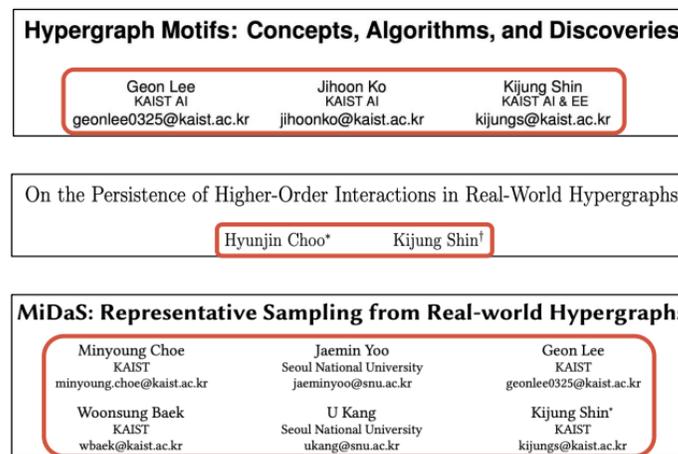
Hypergraph?

Hypergraph

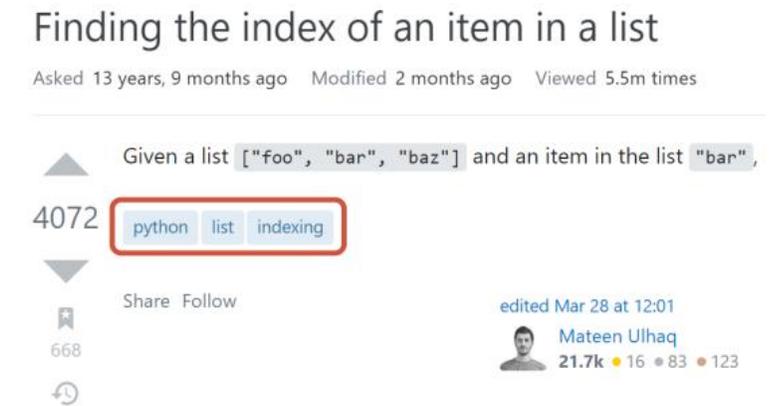
그룹 간 상호작용



같은 채팅방에 존재하는 사람들 (그룹)



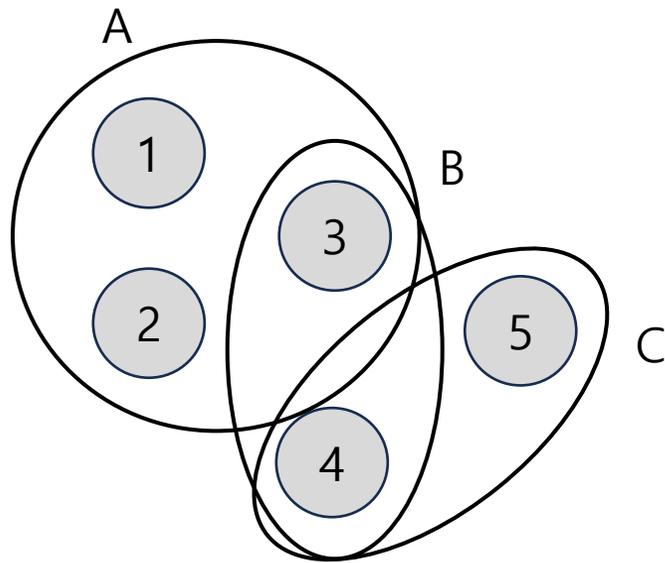
같이 논문을 쓴 사람들 (그룹)



한 게시물의 태그들 (그룹)

Hypergraph

그룹 간 상호작용



채팅방 A 참가자: {1, 2, 3}

채팅방 B 참가자: {3, 4}

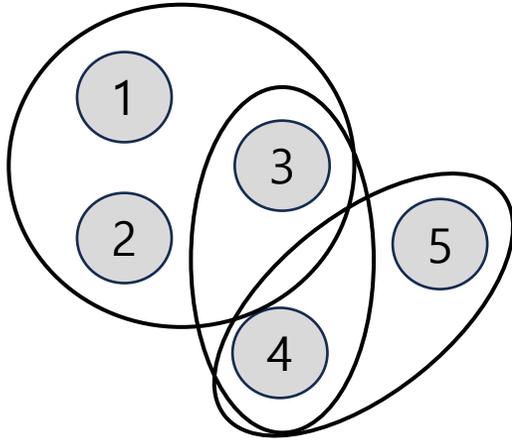
채팅방 C 참가자: {4, 5}

채팅방: Hyperedge (Group)

사용자: Node

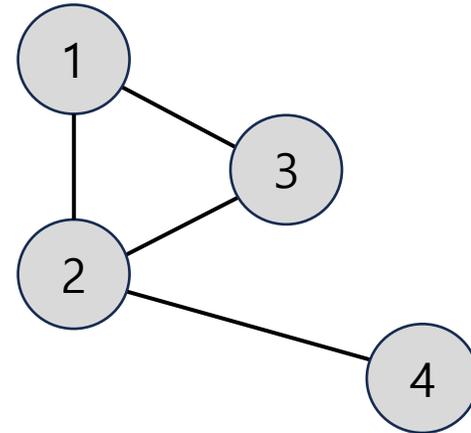
Hypergraph

일반적인 그래프와 비교



Node

Hyperedge (N개의 Node를 연결)

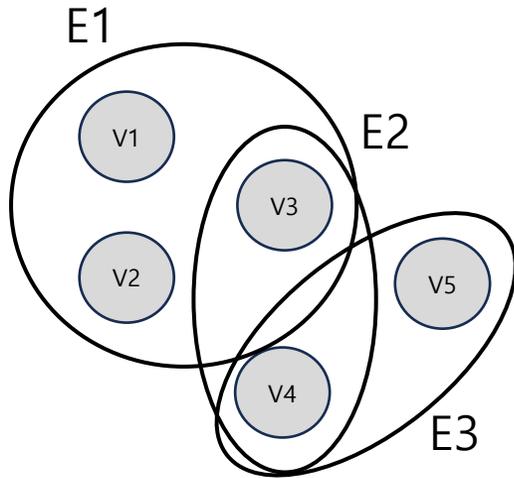


Node

Edge (2개의 Node를 연결)

Hypergraph

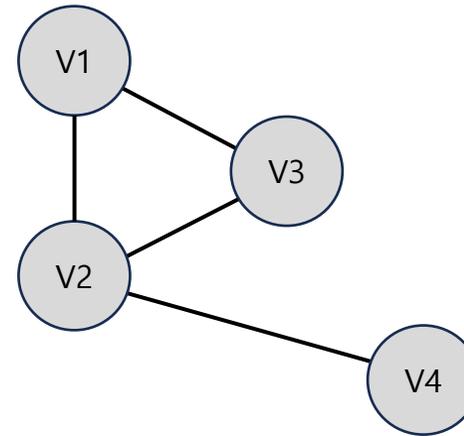
Hypergraph의 표현



Node ($|V| = 5$)
Hyperedge ($|E| = 3$)

| | E1 | E2 | E3 |
|----|----|----|----|
| V1 | 1 | 0 | 0 |
| V2 | 1 | 0 | 0 |
| V3 | 1 | 1 | 0 |
| V4 | 0 | 1 | 1 |
| V5 | 0 | 0 | 1 |

Incidence matrix
 $|V| * |E|$



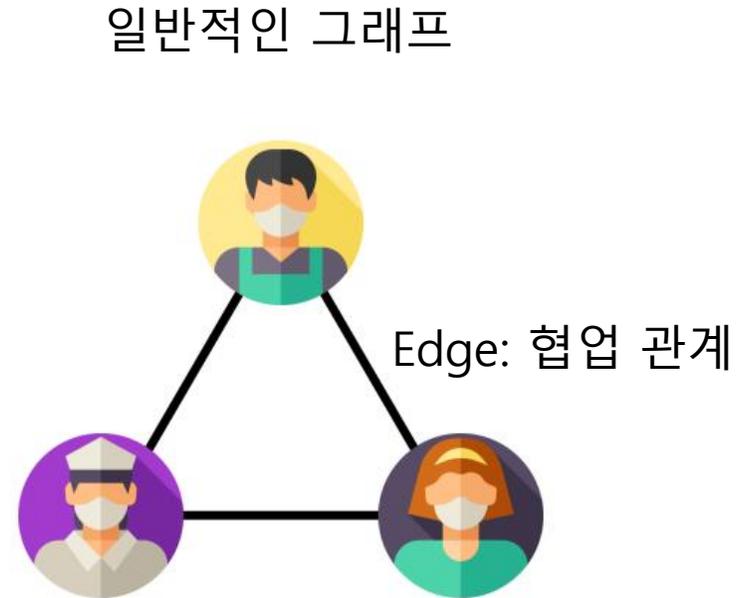
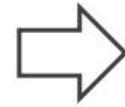
Node ($|V| = 4$)
Edge ($|E| = 4$)

| | V1 | V2 | V3 | V4 |
|----|----|----|----|----|
| V1 | 0 | 1 | 1 | 0 |
| V2 | 1 | 0 | 1 | 1 |
| V3 | 1 | 1 | 0 | 0 |
| V4 | 0 | 1 | 0 | 0 |

Adjacency matrix
 $|V| * |V|$

그래서 왜 필요한가?

일반적인 그래프의 문제점

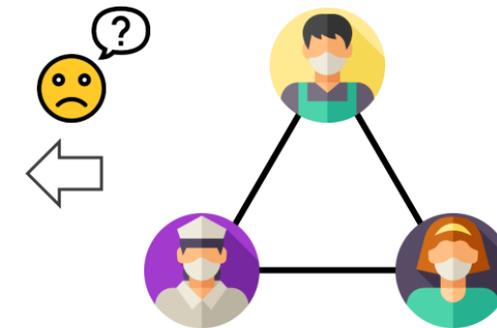
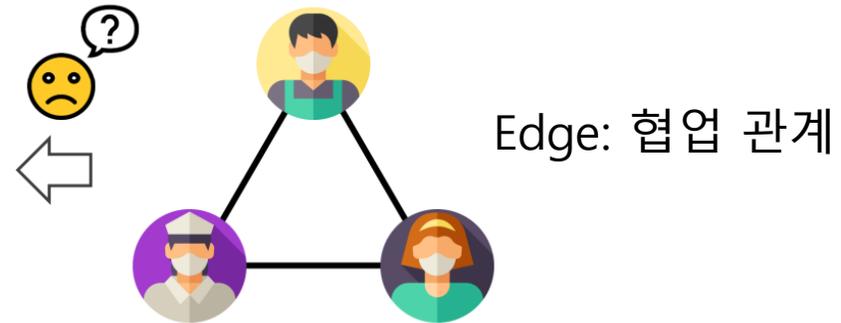


그래서 왜 필요한가?

일반적인 그래프의 문제점



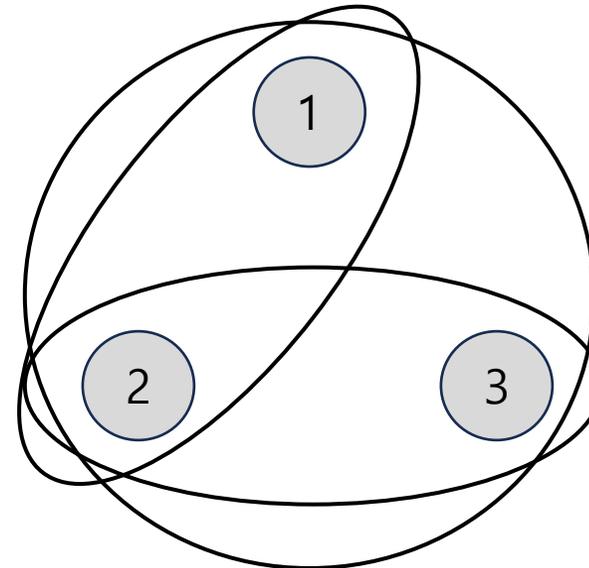
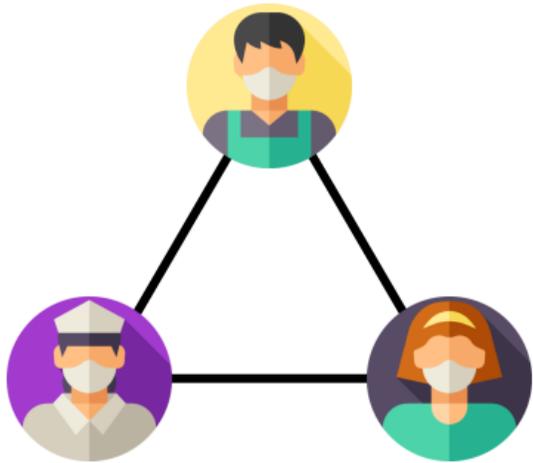
일반적인 그래프



Q. 3명이 함께 논문을 쓴 적이 있나? -> ??

그래서 왜 필요한가?

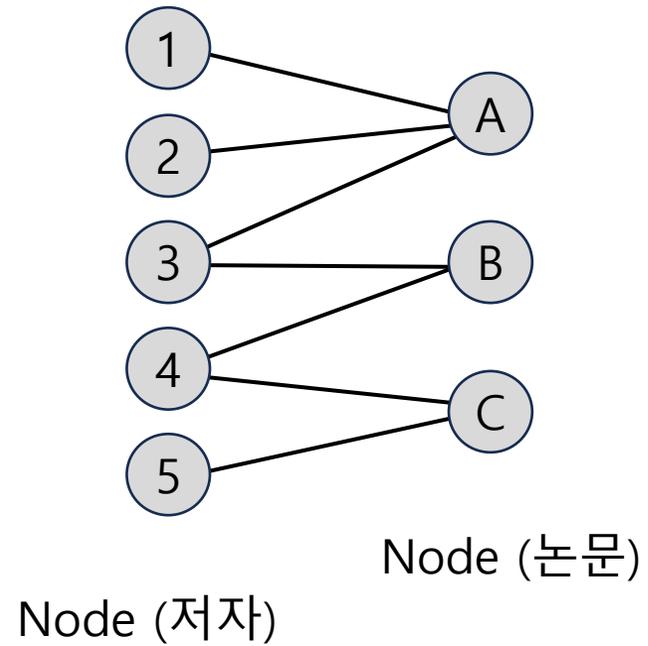
일반적인 그래프의 문제점



Q. 3명이 함께 논문을 쓴 적이 있나? -> 있다!

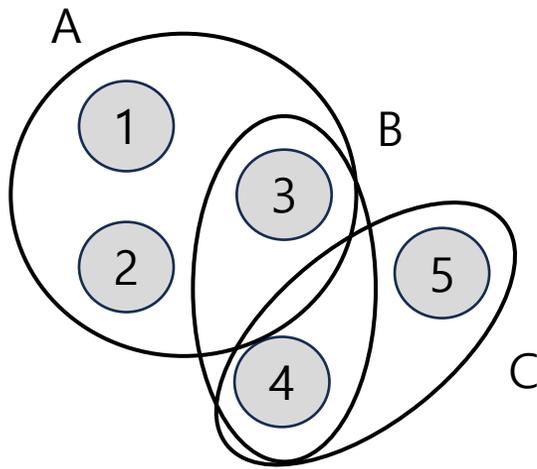
Hypergraph

Bipartite graph



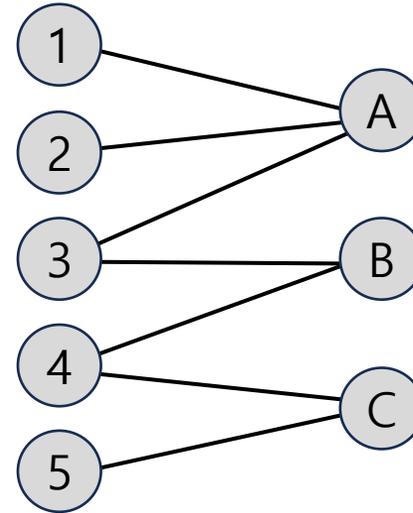
Hypergraph

Hypergraph vs. Bipartite graph



Node (저자)

Hyperedge (논문)



Node (저자)

Node (논문)

Structural Patterns

“What do real-world hypergraphs look like?”

“Given a static hypergraph, how can we analyze its structure?”

-> Structural patterns

그래프를 설명하거나 분석하기 위해 필요

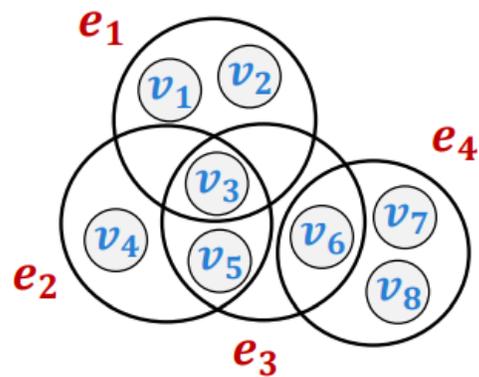
(이 튜토리얼에서는 Generative model이 얼마나 현실적인 Hypergraph를 생성해내는지를 평가하기 위해서 사용됨)

Structural Patterns

Background

Degree of a node: 노드가 포함된 하이퍼엣지의 수

Size of a hyperedge: 하이퍼엣지에 포함된 노드의 수



Hypergraph



Degree of v_5 is 2.
Size of e_2 is 3.

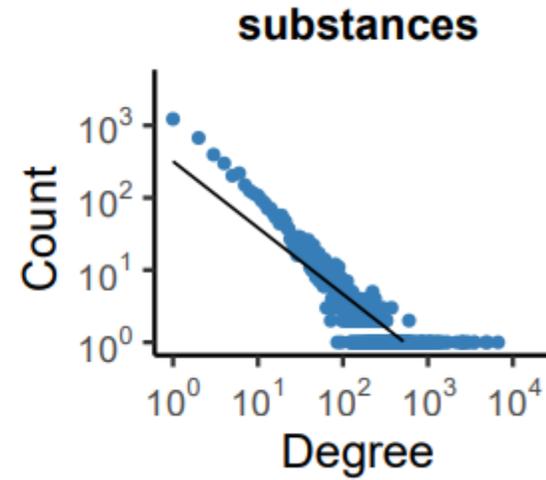
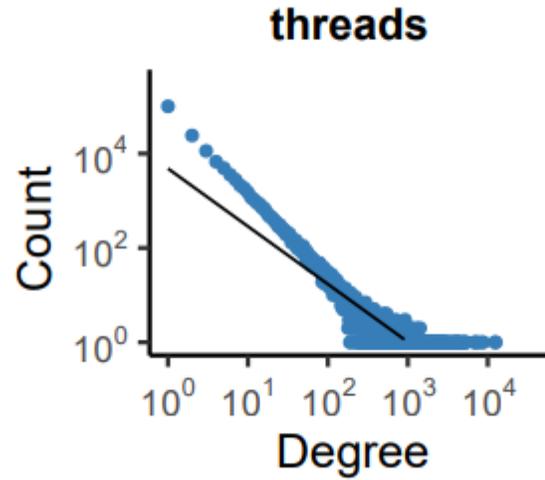
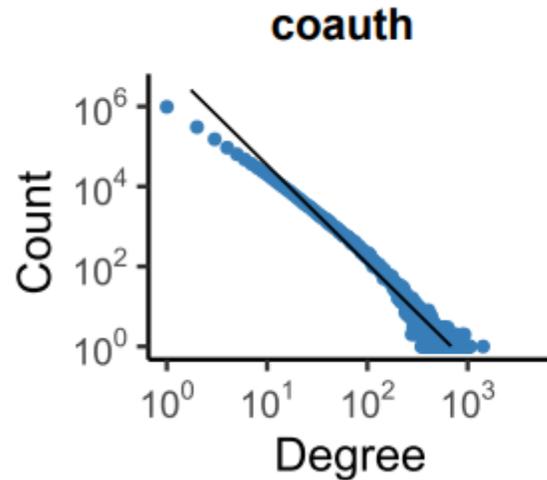
Example

Structural Patterns

차수 분포

각 사람이 얼마나 많은 그룹에 속하는지

Heavy-tailed, 즉 많은 그룹에 속하는 사람일수록 더 많은 그룹에 속하도록 편향됨

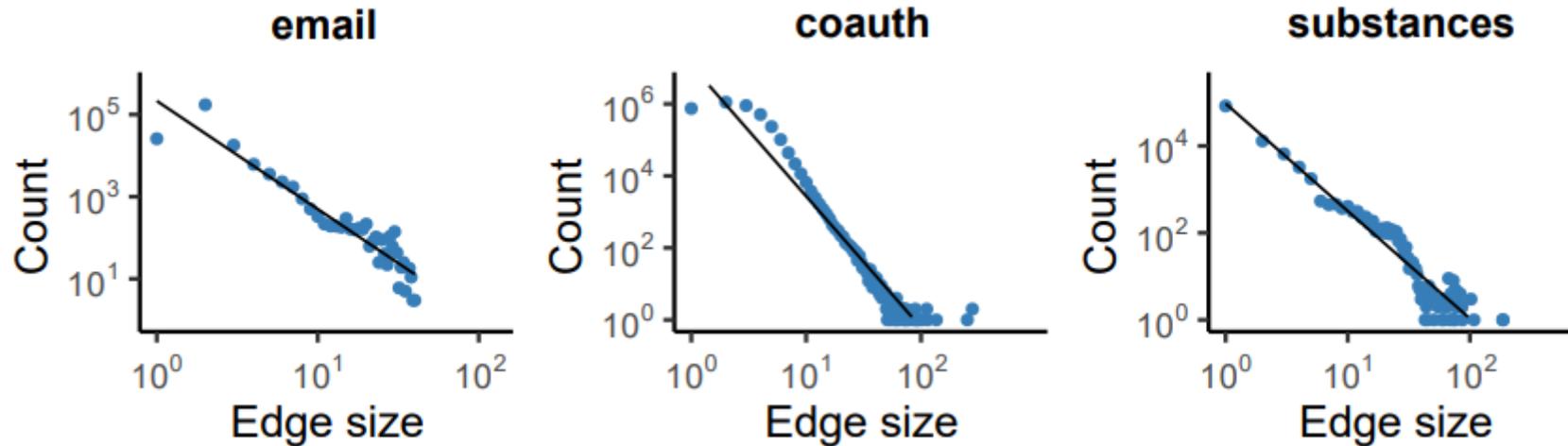


Structural Patterns

Hyperedge Size 분포

각 그룹에 얼마나 많은 사람이 속하는지

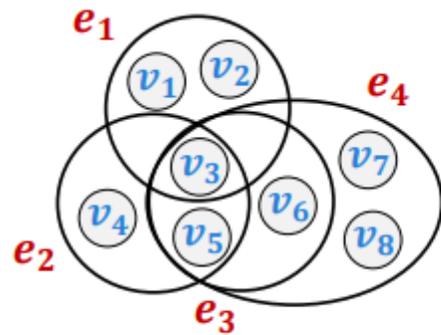
Heavy-tailed, 즉 많은 사람이 속하는 그룹일수록 더 많은 사람이 속하도록 편향됨



Structural Patterns

Pair/Triple Degree의 분포

어떤 두/세 노드가 공통으로 overlap되는 하이퍼엣지의 수



Hypergraph



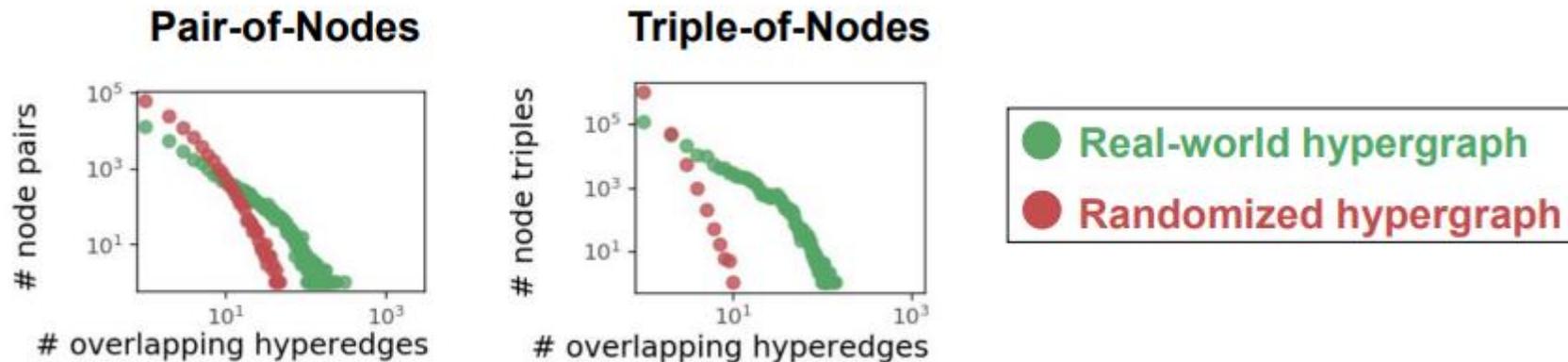
Degree of $\{v_3, v_5\}$ is 3.
Degree of $\{v_3, v_5, v_6\}$ is 2.

Example

Structural Patterns

Pair/Triple Degree의 분포

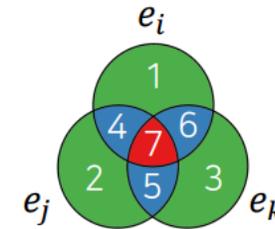
랜덤 데이터에 비해 skew되고 heavy tailed됨



Structural Patterns

Advanced Patterns

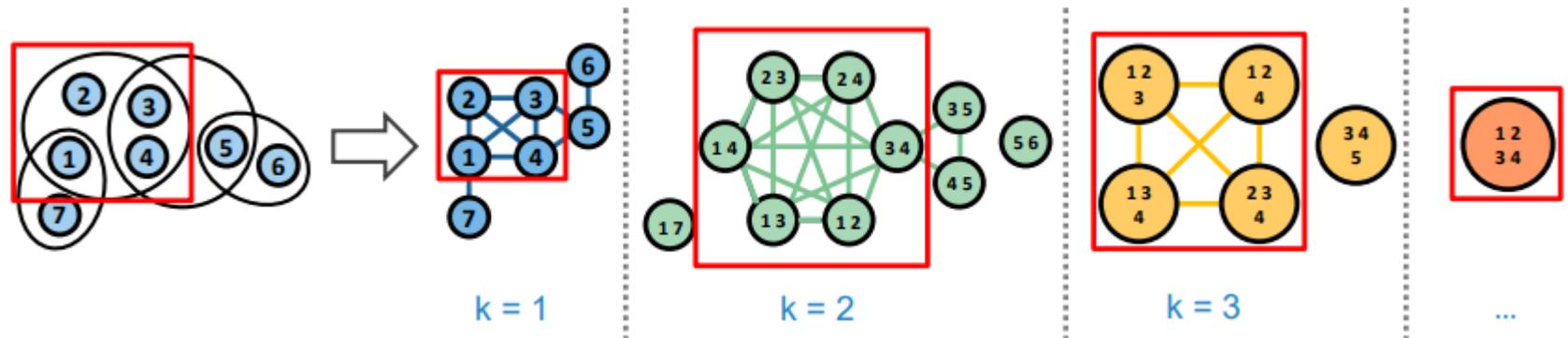
H-motifs: 3개의 하이퍼엣지 간 교집합 구조의 개수를 세서 Graphlet처럼 사용



- | | |
|---------------------------------------|----------------------------------|
| (1) $e_i \setminus e_j \setminus e_k$ | (4) $e_i \cap e_j \setminus e_k$ |
| (2) $e_j \setminus e_k \setminus e_i$ | (5) $e_j \cap e_k \setminus e_i$ |
| (3) $e_k \setminus e_i \setminus e_j$ | (6) $e_k \cap e_i \setminus e_j$ |
| | (7) $e_i \cap e_j \cap e_k$ |

Multi-level decomposition: 여러 개의 레벨로 나눠서, k레벨이라면 노드를 k개씩 묶은 대체 노드들의 하이퍼그래프로 만들어서 사용

등등 많은 것들이 있지만.. 시간 관계상 생략



Generative Models

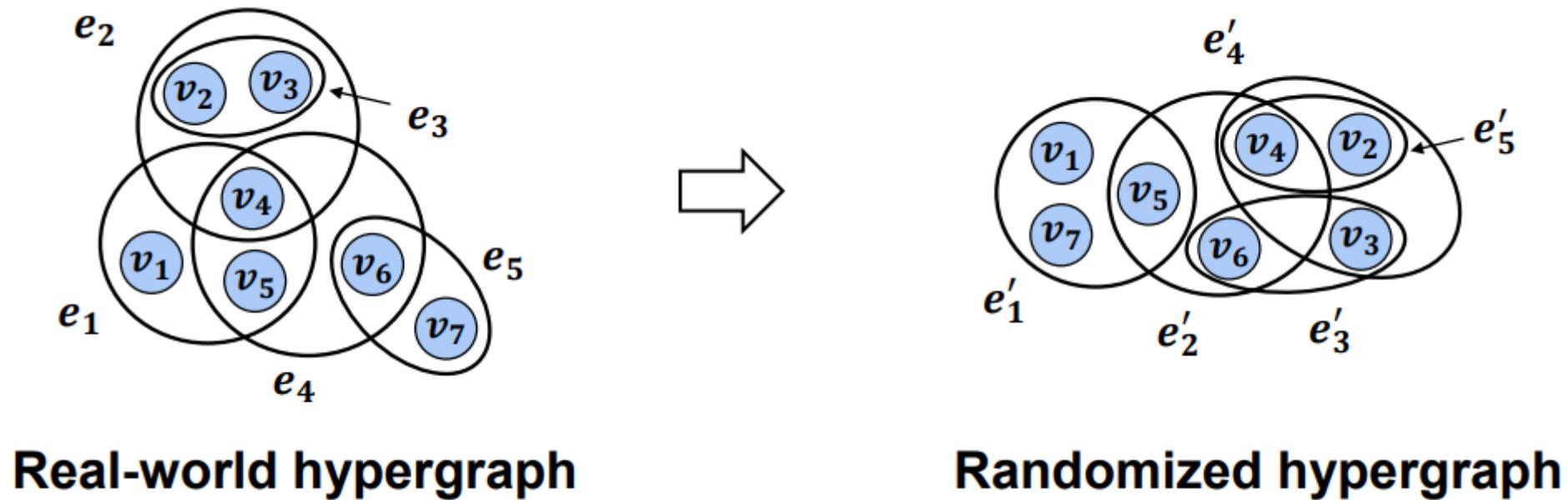
Real-world Hypergraph에 가까운 Hypergraph를 생성하는 것이 목표

생성 모델을 만드는 목적:

- 벤치마크 데이터 생성: 하이퍼그래프에서 동작하는 알고리즘의 벤치마크를 위한 데이터 생성
- 익명화: 민감한 정보를 담고 있는 Real-world 그래프를 유사한 성질을 가진 합성데이터로 변환

Generative Models

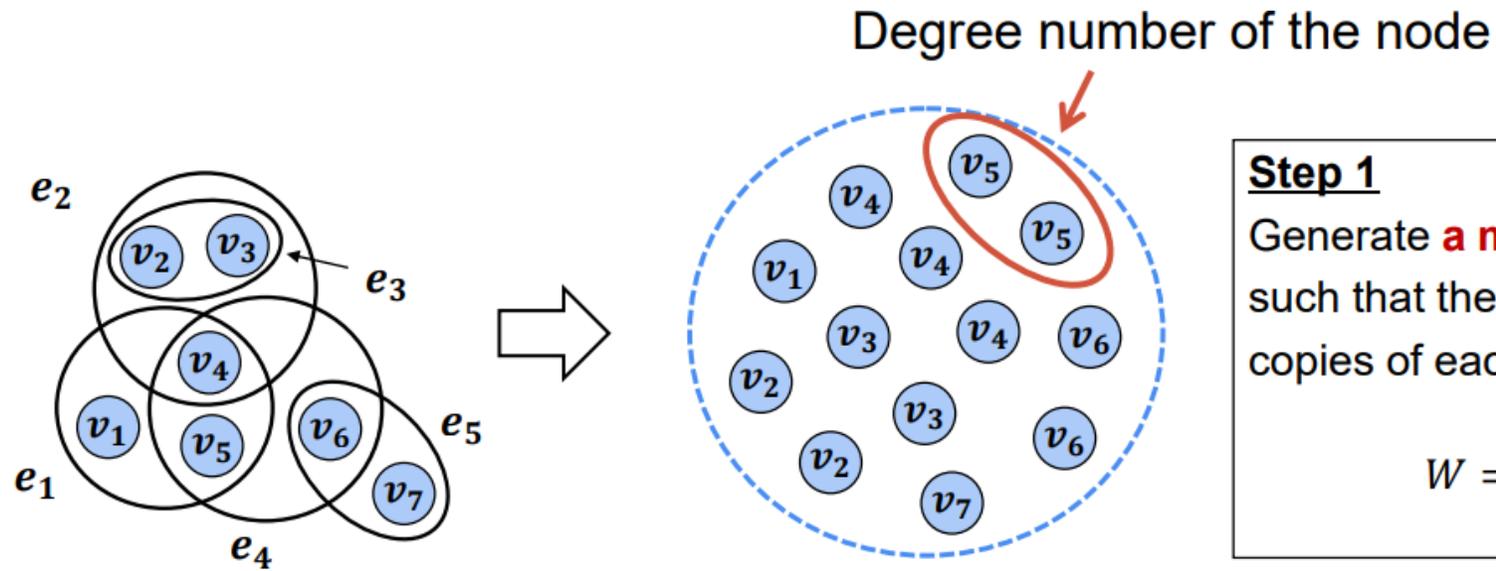
Configuration models: Node Degree와 Hyperedge sizes의 분포를 유지



Generative Models

Step 1. 멀티셋 생성

각 노드마다, 차수만큼 복제해 멀티셋에 넣음 -> 차수 유지



Step 1

Generate **a multiset W** of nodes such that the degree number of copies of each node is contained.

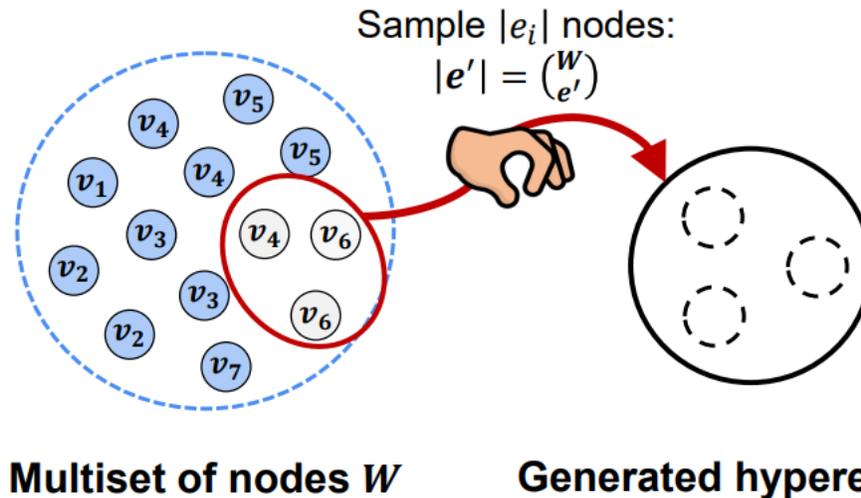
$$W = \bigsqcup_{v \in V} \{v, \dots, v\}$$

Generative Models

Step 2. Hyperedge Sampling

각 Hyperedge마다, Size만큼 샘플링해 가져옴 -> Hyperedge size 유지

Step 2. Hyperedge Sampling



Step 2-1

To generate a hyperedge e'_i , sample $|e_i|$ nodes from multiset W uniformly at random.

$$e'_i = \binom{W}{|e_i|}$$

Step 2-2

Remove the items sampled from W .

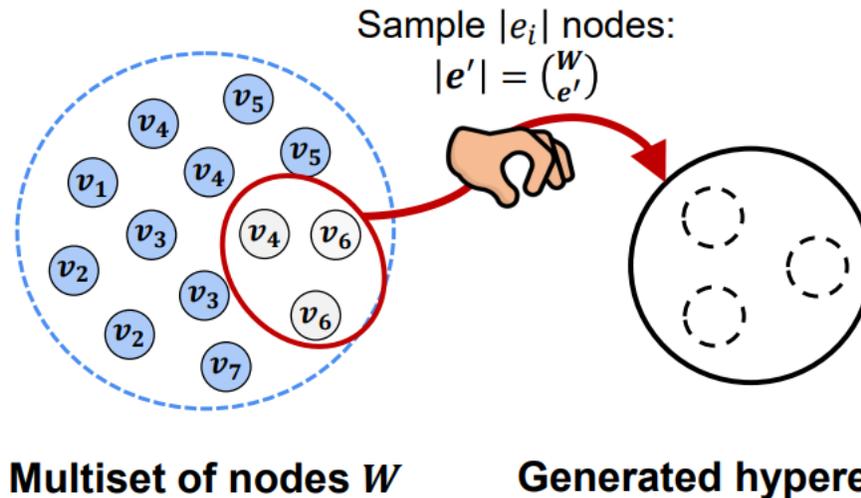
$$W = W \setminus e'_i$$

Generative Models

Step 2. Hyperedge Sampling

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